U.G. Sem ~~B.Sc~~ Part III Physics ~~(Hons)~~ MJC

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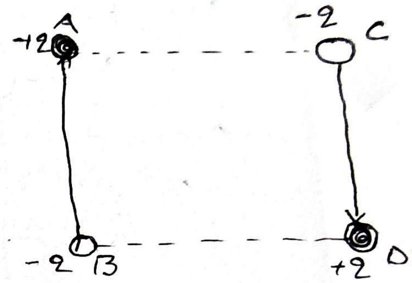
Ques. What do you mean by electric dipole moment and quadrupole moment? obtain an expression for electric potential at a point due to electric quadrupole moment.

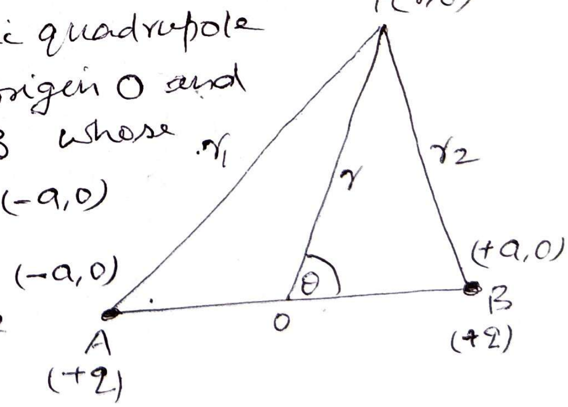
Ans :- Dipole Moment :→ when two equal and opposite electric charges are on the least distance constitute electric dipole. Dipole moment is the product of charge of the pole and the distance between ~~two~~ both poles.



Let A and B point consists of +q and -q electric charge. which is on dl distance from one to another. Then electric dipole moment $\vec{p} = q \cdot dl$ which is vector.

Quadrupole moment :→ when two dipoles are in opposite direction and displaces by one another, a quadrant structure formed and is called quadrupole moment. Let AB & CD two dipole are in parallel to one another and in apposite directions. That arrangement is called two demensional quadrupole. Again Let AB and CD are on the same line as its moment is derected in apposite direction, that arrangement is called an amplitude quadrupole. Thus $\vec{AB}$ and $\vec{CD}$ are the mement of dipole or dipole moment $\vec{p}$ which is among in opposite direction, called electric quadrupole moment, which makes an electric quadrupole.

Expression for electric Potential at any point due to an electric quadrupole :→



Let AOB is an electric quadrupole and a point charge $-2q$ is on its origin O and whose +q charge is on the point A and B whose co-ordinates relative to O are $(+a,0)$ & $(-a,0)$ ~~rel~~ respectively, i.e $AB = 2a$. Then the electric potential is to determine on point P at a distance r from O.

Due to AOB quadrupole whose polar co-ordinate is (r, θ). Connect the PA, PO and PB, $OP = r$ and $\angle AOB = \theta$. Let $PA = r_1$ and $PB = r_2$. AB is very small in comparison to OP, i.e. $a << r$

Now an electric potential at point P due to $+q$ charge on the point A $= q/r_1$, an electric Potential at point P due to $+q$ charge on B point $= q/r_2$ and an electric potential at point P due to point P of $-2q$ charge on O point $= \left(-\frac{2q}{r}\right)$

Therefore, total electric potential P point due to quadrupole

$$V = \left(\frac{q}{r_1} + \frac{q}{r_2} - \frac{2q}{r}\right) \quad \text{------------- (I)}$$

Now in $\triangle$ APO, by cosine law we have

$$r_1^2 = r^2 + a^2 + 2ar\cos\theta$$

$$= r^2\left[1 + 2\frac{a}{r}\cos\theta + \frac{a^2}{r^2}\right]$$

$$\therefore r_1 = r\left[1 + 2\frac{a}{r}\cos\theta + \frac{a^2}{r^2}\right)^{\frac{1}{2}}$$

Similarly in $\triangle$ POB, by cosine law, we have

$$r_2^2 = r^2 + a^2 - 2ar\cos\theta$$

$$= r^2\left[1 - \frac{2a}{r}\cos\theta + \frac{a^2}{r^2}\right]$$

$$\therefore r_2 = r\left[1 - \frac{2a}{r}\cos\theta + \frac{a^2}{r^2}\right)^{1/2}$$

from (I)

$$V = \frac{q}{r}\left[\left(1 + 2\frac{a}{r}\cos\theta + \frac{a^2}{r^2}\right)^{-\frac{1}{2}} + \left(1 - 2\frac{a}{r}\cos\theta + \frac{a^2}{r^2}\right)^{-\frac{1}{2}} - 2\right] \quad \text{--- (II)}$$

By expanding the term $\left[1 + 2\frac{a}{r}\cos\theta + \frac{a^2}{r^2}\right)^{-1/2}$ in eqn (II) using Binomial theorem

$$\left(1 + 2\frac{a}{r}\cos\theta + \frac{a^2}{r^2}\right)^{-\frac{1}{2}} = \left[1 + \frac{1}{2}\left(\frac{2a}{r}\cos\theta + \frac{a^2}{r^2}\right) + \frac{1}{2}\cdot\frac{3}{4}\left(\frac{2a}{r}\cos\theta + \frac{a^2}{r^2}\right)^2 \cdots\cdots\right]$$

$$= \left[1 - \frac{1}{2}\left(\frac{2a}{r}\cos\theta + \frac{a^2}{r^2}\right) + \frac{3}{8}\left(\frac{4a^2}{r^2}\cos^2\theta + \frac{4a^3}{r^3}\cos\theta + \frac{a^4}{r^4}\right)\right]$$

$$= \left[1 - \frac{a}{r}\cos\theta + \frac{a^2}{2r^2} + \frac{3}{2}\cdot\frac{a^2}{r^2}\cos^2\theta + \frac{3a^3}{2r^3}\cos\theta + \frac{3}{8}\frac{a^4}{r^4}\right]$$

$$= \left[1 - \frac{a}{r}\cos\theta + \frac{a^2}{2r^2}(3\cos^2\theta - 1) + \frac{3a^3}{2r^3}\cos\theta\right]$$

Similarly $\left(1 - \frac{2a}{r}\cos\theta + \frac{a^2}{r^2}\right)^{-1/2}$

$$= \left[1 + \frac{a}{r}\cos\theta + \cancel{\frac{a^2}{2r}}\,\frac{a^2}{2r^3}(3\cos^2\theta - 1) - \frac{3a^3}{2r^3}\cos\theta\right]$$

Therefore we have

$$V = \frac{q}{r}\left[1 - \frac{q}{r}\cos\theta + \frac{a^2}{2r^2}(3\cos^2\theta - 1) + \frac{3a^3}{2r^3}\cos\theta + 1 + \frac{a}{r}\cos\theta + \frac{a^2}{2r^2}(3\cos^2\theta - 1) - \frac{3a^3}{2r^3}\cos\theta - 2\right]$$

$$= \frac{q}{r}\left[2\frac{a^2}{2r^2}(3\cos^2\theta - 1)\right]$$

$$= \frac{qa^2}{r^3}(3\cos^2\theta - 1)$$

Thus an electric potential at point P due to quadrupole

$$V = \frac{qa^2}{r^3}(3\cos^2\theta - 1)$$